

# Mathematical understanding

Learning in this area should include an appropriate balance of focused subject teaching and well-planned opportunities to use, apply and develop knowledge and skills across the whole curriculum.

## Curriculum aims

This area of learning contributes to the curriculum aims for all young people to become:

- successful learners who enjoy learning, make progress and achieve
- confident individuals who are able to live safe, healthy and fulfilling lives
- responsible citizens who make a positive contribution to society.

## Why is this area of learning important?

Mathematics introduces children to concepts, skills and thinking strategies that are useful in everyday life and support learning across the curriculum. It provides a way of handling information and making sense of data in an increasingly digital world.

Children draw great satisfaction from using their mathematical skills to solve a problem, often gaining a sense of wonder and excitement when it leads them to an unexpected discovery or allows them to make new connections. As their confidence grows, they are able to build on their natural inquisitiveness and creativity by investigating patterns, conjectures and generalisations and trying different methods to provide solutions.

Mathematics helps children make sense of the numbers, patterns and shapes they see in the world around them. With the logical reasoning and systematic thinking they learn in mathematics, children can solve problems, make estimates, use evidence to construct persuasive arguments and explore 'what if?' questions using mathematical models.

The precise and unambiguous nature of mathematical statements introduces children to a powerful way of communicating. They learn to explore and explain their ideas using symbols, diagrams and spoken and written language. They start to discover how mathematics has developed over time and contributes to our economy, society and culture.

## Explanatory text:

## 1. Essential knowledge

Children should build secure knowledge of the following:

- a. the ways that numbers are used and what they represent
- b. how numbers can be used for quantification and comparison and applied in different contexts
- c. how to use geometry to explore, understand and represent shape and space
- d. how likelihood and risk can be understood, quantified and used in everyday life
- e. the range of ways mathematics can be used to solve practical problems, model situations, make sense of data and inform decision making.

## 2. Key skills

These are the skills that children need to learn to make progress:

- a. generate and explore ideas and strategies, pursue lines of mathematical enquiry and apply logic and reasoning to mathematical problems
- b. make and test generalisations, identify patterns and recognise equivalences and relationships<sup>1</sup>
- c. develop, select and apply a range of mental, written and ICT-based methods and models to estimate, calculate, classify, quantify, order and compare
- d. communicate ideas and justify arguments using mathematical symbols, diagrams, images and language
- e. interpret findings, evaluate methods and check outcomes.

### Explanatory text:

Teachers will continue to find the numeracy framework a significant basis for planning teaching.

1. This includes families of equivalent fractions; the inverse relationship between addition and subtraction.

### 3. Breadth of learning

- a. Children should experience mathematics as a creative activity and be introduced to its role in the world around them. They should develop their mathematical understanding through focused, practical, problem-solving activities in mathematical, cross-curricular and real world contexts. Children should have opportunities to meet with people who use mathematics in their work. They should also use a wide range of practical resources, including ICT. Working on their own and with others they should explore ideas and pursue lines of mathematical enquiry.
- b. Children should be taught to think and work logically, creatively and critically as they solve problems, make sense of information, manage money, assess likelihood and risk, predict outcomes and construct conjectures and arguments.
- c. They should be taught to visualise quantities, patterns and shapes and develop strategies for working things out in their head as well as on paper and using ICT. They should make choices about the strategies they use to solve problems, based on what they know about the efficiency and effectiveness of different approaches. They should also be introduced to the mathematical language they need to explain, refine and evaluate their own and others' work.

#### Explanatory text:

## 4. Curriculum progression

The overall breadth of learning should be used when planning curriculum progression. Children should be taught:

EARLY <sup>2</sup>	MIDDLE	LATER
<b>Number and the number system</b>		
<p>E1. to estimate the number of objects and count them, recognising conservation of number</p> <p>E2. read, write and order numbers to 100 and beyond using a range of representations<sup>3</sup></p> <p>E3. to explore and explain patterns<sup>4</sup>, including number sequences in the counting system</p> <p>E4. to group, match, sort, partition and recombine numbers, developing an understanding of place value.</p>	<p>M1. to understand and interpret negative numbers, simple fractions<sup>16</sup>, large numbers and tenths, written as decimals, in practical and everyday contexts</p> <p>M2. to generate and explore a range of number patterns, including multiples<sup>17</sup></p> <p>M3. to make and test general statements about numbers, sort and classify numbers and explain methods and findings</p> <p>M4. to approximate numbers, including rounding<sup>18</sup>, and understand when that can be useful</p> <p>M5. about the representation of number in different contemporary cultures<sup>19</sup>.</p>	<p>L1. to use decimals up to three decimal places in measurement contexts</p> <p>L2. to understand and use the equivalence of families of fractions and their decimal representation when ordering and comparing</p> <p>L3. to explore number patterns and properties<sup>28</sup>, and represent them using graphs, simple formulae and ICT<sup>29</sup></p> <p>L4. about the development of the number system<sup>30</sup></p> <p>L5. to interpret computer and calculator displays and round to an appropriate level of accuracy.</p>
<p><b>Explanatory text:</b></p> <p>2. Each area of learning should build on children's experiences and development in the Early Years Foundation Stage to ensure continuity of curriculum provision and their continuing progress.</p> <p>3. For example, number lines, number squares, structural apparatus.</p> <p>4. This includes additive number sequences, such as counting in groups of e.g. 2, 5 or 10, odds and evens; and relationships between numbers, e.g. the sum of two odd numbers is always even. Using calculators to explore number patterns and properties is important here.</p>	<p><b>Explanatory text:</b></p> <p>16. Simple fractions include half, third, quarter, fifth, tenth, two thirds and three quarters.</p> <p>17. Using ICT for changing values and exploring in a spreadsheet model.</p> <p>18. For example rounding to the nearest ten, hundred and thousand.</p> <p>19. For example Arabic, Chinese and Indian numerals.</p>	<p><b>Explanatory text:</b></p> <p>28. This includes factors, primes and square numbers.</p> <p>29. Changing variables and rules in spreadsheet models; using graphing software.</p> <p>30. For example, the Roman and Egyptian number systems do not use a place value; Babylonian numbers and Mayan numbers use base 60 and base 20 respectively; Greeks explored square and triangle numbers.</p>

## Number operations and calculation

- E5. a range of strategies for combining, partitioning, grouping and sharing (including doubling and halving) and increasing and decreasing numbers, to solve practical problems<sup>5</sup>.
- E6. to use number bonds to ten to add and subtract mentally<sup>6</sup> numbers with one or two significant figures
- E7. to represent addition and subtraction as number sentences including finding missing numbers and understanding the equals sign<sup>7</sup>.

### Explanatory text:

5. This lays the foundations for understanding number operations.
6. For example  $700 + 300 =$ ;  $60 + \square = 100$ ;  $57 + 33 =$ ;  $57 - 8 =$ ; this develops their understanding of the inverse relationship between addition and subtraction.
7. For example  $3 + 1 = 1 + 3$ ;  $3 + 1 = \square + 2$ ;  $3 + 1 = 5 - \square$ .

- M6. to compare two numbers by finding the difference between them<sup>20</sup>
- M7. to use the relationship between addition and subtraction<sup>21</sup> and addition and multiplication to understand and generate equivalent expressions<sup>22</sup>
- M8. to use simple fractions to find fractional parts and express proportions
- M9. to select from a range of mental strategies for the addition and subtraction of numbers with two significant figures
- M10. to understand division as grouping and as sharing and solve division problems using multiplication facts<sup>23</sup>
- M11. to visualise and understand multiplication represented as an array, record multiplication as number sentences and solve problems using multiplication facts
- M12. to use estimation to find approximate answers to calculations<sup>24</sup>, to record calculations and check answers and methods.

### Explanatory text:

20. For example finding how much the temperature changed.
21. For example, since  $54 + 37 = 91$ ,  $91 - 37 = 54$  and  $91 - 54 = 37$ .
22. For example  $3 \times 13 = 3 \times 10 + 3 \times 3$ ;  $5 \times 19 = 5 \times 20 - 5 \times 1$ .
23. Multiplication facts should include 2, 3, 4, 5 and 10.
24. For example to estimate the cost of an apple sold in a pack of four or to recognise that  $296 + 735$  will be approximately 1000.

- L6. to use simple ratio<sup>31</sup>, percentages and fractions to compare numbers and quantities and solve problems
- L7. to extend their knowledge of multiplication facts to  $10 \times 10$  and use them to solve multiplication and division problems
- L8. to understand and use different models of division, including interpreting the outcome of a division calculation, in relation to the context, where the answer is not a whole number
- L9. to recognise and use the relationship between fractions and division and represent division as number sentences<sup>32</sup>
- L10. to recognise and use the relationships between addition, subtraction, multiplication and division
- L11. to develop a range of strategies for calculating and checking, including using a calculator or computer efficiently
- L12. to solve multi-step problems involving more than one operation.

### Explanatory text:

31. For example 45 is three times greater than 15, they are in the ratio 3:1.
32. For example  $325 \div 5 = (300 + 25) \div 5 = 300 \div 5 + 25 \div 5$  or  $325/5 = 300/5 + 25/5$ .

EARLY<sup>2</sup>

## MIDDLE

## LATER

## Money

- E8. to use coins of different values and recognise the equivalence of different combinations of coins<sup>8</sup>
- E9. to compare and order costs of different items.

- M13. to record amounts of money using pounds and/or pence, converting between them as appropriate
- M14. how to handle amounts of money in the contexts of shopping, saving up and enterprise activities<sup>25</sup>.

- L13. to solve problems related to borrowing, spending and saving<sup>33</sup>
- L14. to understand and convert between different currencies
- L15. how to manage money<sup>34</sup> and prepare budgets for events, including using spreadsheets.

## Measures

- E10. to compare and order objects and events<sup>9</sup>
- E11. to create and use whole number scales<sup>10</sup> to measure.

- M15. to recognise when length and capacity are conserved
- M16. to use standard units to estimate measures and to measure with appropriate accuracy
- M17. to recognise and use equivalent representations of time
- M18. to measure angles using fractions of turn and right angles
- M19. to explore the development of different measuring systems, including metric and Imperial measures.

- L16. to recognise when area, volume and mass are conserved
- L17. to convert between units within the metric system
- L18. to use an angle measurer to measure angles in degrees
- L19. to solve problems involving time and time intervals, including time represented by the 24-hour clock
- L20. use decimal calculations to solve problems with measures.

**Explanatory text:**

8. Including in the context of buying and selling involving role play.
9. This includes mass and length, for example answering questions such as 'which is heaviest?' or 'which is longest?'
10. Number scales include standard and non-standard units.

**Explanatory text:**

25. For example to find and compare unit costs of items that are sold in multiple unit quantities.

**Explanatory text:**

33. This includes using and interpreting information from external sources and making decimal calculations.
34. This includes using the context of enterprise activities where children need to work out a range of budgetary options, developing awareness of profit and loss.

EARLY<sup>2</sup>

## MIDDLE

## LATER

## Geometry

- E12. to identify, group, match, sort and compare common shapes<sup>11</sup> using geometric properties<sup>12</sup>.
- E13. to identify, reproduce and generate geometric patterns including the use of practical resources and ICT
- E14. to generate instructions for movement<sup>13</sup>.

- M20. to recognise symmetry properties of 2D shapes and patterns
- M21. to make simple scalings<sup>26</sup> of objects and drawings
- M22. to understand and use angle as the measure of turn
- M23. to understand perimeter as a length and to find the perimeter of rectangles and other shapes
- M24. to create sequences of instructions using ICT, including generating symmetric and repeating geometric patterns.

- L21. to use and make maps, scale models and diagrams for a purpose
- L22. to understand area as the space enclosed by a perimeter on a plane, and find areas of rectangles and related shapes<sup>35</sup>
- L23. to solve practical problems involving 3D objects<sup>36</sup>
- L24. to visualise geometric objects<sup>37</sup> and to recognise and make 2D representations of 3D shapes
- L25. to create and refine sequences of instructions, using ICT to construct and explore geometric patterns and problems<sup>38</sup>
- L26. to explore aspects of geometry to find out about its origins<sup>39</sup>, and its use in different cultures, religions, art and architecture<sup>40</sup>.

## Explanatory text:

11. Common shapes include triangle, square, rhombus, rectangle, oblong, kite, parallelogram, circle, cube, prism, pyramid, cylinder, cone, and sphere.
12. Geometric properties include edges, vertices, faces, straight, curved, closed and open.
13. For example using a programmable toy or describing a familiar journey including change of direction.

## Explanatory text:

26. Simple scales include half, twice and ten times.

## Explanatory text:

35. This includes triangles and shapes that are made up of triangles and rectangles including the surface area of 3D objects.
36. This includes developing understanding of the volume of cuboids by solving problems such as what is the smallest possible box to hold six smaller boxes?
37. This includes imagining what something will look like in different orientations.
38. This should include use of procedures to improve efficiency of sequences.
39. For example Greek architecture and discoveries, stone circles and Pyramids.
40. For example Islamic patterns, Japanese temple art, Rangoli patterns, modern art and ancient and modern architecture.

EARLY<sup>2</sup>

## MIDDLE

## LATER

## Statistics

- E15. to generate and explore questions that require the collection and analysis of information
- E16. to collect, group, match, sort, record and represent information<sup>14</sup> for a purpose and store it using ICT<sup>15</sup>
- E17. to interpret and draw conclusions from information they have collected<sup>15</sup>.

- M25. to collect and structure information using ICT so that it can be searched and analysed, including using appropriate field headings and data types<sup>27</sup>
- M26. to use frequency diagrams and bar charts to represent and record information
- M27. to interpret their own and others' data.

- L27. how statistics are used in society today<sup>41</sup>
- L28. to use mean, mode, median and range to summarise and compare data sets
- L29. to use data to assess likelihood and risk and develop an understanding of probability through computer simulations, games and consideration of outcomes of everyday situations
- L30. to discuss, sort and order events according to their likelihood of occurring
- L31. to answer questions or test hypotheses by using ICT to collect, store, analyse and present data<sup>42</sup>
- L32. to use ICT to represent data<sup>43</sup> on a scattergraph, and proportional data<sup>44</sup> in a pie chart in order to explore possible relationships and interpret the findings<sup>45</sup>.

## Explanatory text:

14. This includes using Venn and Carroll diagrams, simple frequency diagrams and simple data handling software to create tables and graphs.
15. Including outcomes from using simple data-handling software.

## Explanatory text:

27. Analysis should include discussion about 'reasonableness' of outcomes.

## Explanatory text:

41. For example statistics are used to inform the public about how the local council spend their money, to monitor safety in factories, to inform decisions about whether to install traffic lights, or to decide what stock to order.
42. For example using data types including text, number, date, currency, yes/no and error checking through inspecting outcomes.
43. For example height and weight for a chart on a child's development.
44. Proportional data means data where fractions of the population are represented, such as how a council spends its budget, or how all the children in a class travel to school.
45. This should include understanding how these diagrams work and choosing the appropriate representation to present the data.

## 5. Cross-curricular studies

Children should have opportunities:

- a. to develop and apply the skills of literacy, numeracy and ICT, particularly through the use and practice of the language of mathematics in discussions, writing and role play. This develops their ability to articulate their ideas, negotiate meanings, record their work logically and accurately and explain their reasoning when solving problems that require the use of mathematics. They use ICT to explore geometric properties and develop visual imagery; access information on the internet; explore possibilities when using mathematical models; and present mathematical information and outcomes.
- b. to extend their personal, emotional and social development, particularly through developing logical argument, systematic organisation, pattern recognition, generalisation and critical thinking. Through learning to work collaboratively with others they develop empathy and learn to respect others' ideas.
- c. to enhance their mathematical understanding through making links to other areas of learning and to wider issues of interest and importance, in particular through appreciating the power of mathematics when collecting and making sense of experimental data in scientific and technological understanding; exploring the geometry of art based on patterns in art and design; and creating timelines, using plans and maps and using data to analyse a real problem in the community in historical, geographical and social understanding.